Auctioning Corporate Bonds:

A Uniform-Price with Investment Mandates

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Abstract

There has been a rapid growth in the use of investment mandates for the management of fixed-income assets. In this paper, we examine how the limits set in investment mandates can affect the bidding strategy during the issuance of a corporate bond. We apply the uniform-price auction and prove the existence of symmetric Bayesian Nash equilibrium. Under the presence of an exogenous secondary market, the bidding is reduced when there is an expectation for higher yields on resale. Also, we result that the number of participating investors always affects inversely the bidding strategy, while the issue rate not at all. We provide the necessary comparative statics.

Keywords- Bond Market, Auctions, Market Structure, Market Design, Risk Limits, Budget Limits

JEL Classification - D44, D47, D82, D81, G23

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1 Introduction

Over the last decade the global market size of corporate bonds has more than tripled yet with a decline in the overall bond credit quality (25% in 2019 with non-investment grade) in combination with the longer maturities. The new environment triggered the enforcement of regulatory quantitative limits and self-imposed rating-based investment mandates and policies. For these reasons, there has been a large shift in passive investment management[1] with major institutional investors using external credit ratings for their investment decisions and asset allocation [Appel et al. 2016]. For example, corporate bond ETFs[2] which typically use passive investment tools have reached from USD 32 billion in 2008 to USD 420 billion in 2018 [Çelik et al. 2020].

This trend has also been expanded in the primary market of corporate bonds, where the demand for the new issuance may be included in bond indexes if certain criteria are satisfied. Dathan et al. [2020] show that issuers exploit this passive demand by issuing index-eligible bonds with favorable characteristics and a higher passive demand increases issuers' propensity towards a new issuance.

Surprisingly, more and more investors complain that the access in the primary markets is restricted only to "flippers"[3] supporting that "allocations always come down to favors" [Cornelli and Goldreich 2001]. Resting upon this

[1]These strategies refer to a buy-and-hold portfolio strategy for long-term horizons. They are implemented by benchmarking certain market indexes (e.g. Barclays U.S. Corporate Bond Index,i Shares Short-Term Corporate Bond ETF).

[2]Passive investment vehicles which track various market indexes.

[3]Immediately resale or "flip" the bonds to other broker-dealers at a profit (https://www.sec.gov/news/press-release/2020-159).

premise the Securities Exchange Commission (SEC) has launched an investigation on how large financial institutions handle the allocation of debt during the issuance and penalized in 2020 a big institution for such violation[4]. To this extent, the Securities and Exchange Board of India (Sebi) proposed in 2016 a uniform-price auction for the pricing of corporate bonds which would help deepening the market[5].

The common practice for the pricing of newly issued corporate bonds is an open-price process, called book-building. Briefly, the issuer assigns to the underwriter the competitive sale and the efficient allocation of the new issuance. The underwriter markets the offering to investors, asking a non-binding indication of interest (IOIs) [Iannotta 2010; Nikolova et al. 2020]. This pre-market information helps the underwriter to measure the demand and adjust the offering's price and coupon if needed. A key feature of the process is that the allocation is left in the discretion of the underwriter with the IOIs often being cheaper than the final price[6]. Due to this pre-play communication which reveals investors' valuations, the issuance's allocation weakly corresponds to bidding while the yield not at all [Habib and Ziegler 2007].

In this study, we develop an exploitative model for the pricing of corporate bonds in the primary market. We adopt the mechanism of a common-value uniform-price auction, which allows each investment manager to act as a bidder

[4]https://www.ft.com/content/55406aea-a30a-11e3-ba21-00144feab7de

https://www.thetradenews.com/ubs-agrees-to-pay-10-million-to-the-sec-to-resolve-bonds-sale-violation-charges/

[5]https://economictimes.indiatimes.com/mf/mf-news/sebi-proposes-uniform-pricing-for-debt-securities/articleshow/64200207.cms?from=mdr

[6]Commission Expert Group on Corporate Bonds, Analysis of European Corporate Bond Markets, November 2017

and submit sealed-bid demand schedules. Each demand schedule specifies the desired share over a fully divisible bond, at different yield levels complying to bounds set by the book-runner.

Uniform-price auctions have been widely used in U.K. and U.S. for the selling of Treasury securities, with much debate in auction theory to be around the optimal choice between uniform-price and discriminatory auctions. However, neither empirical research [Nyborg and Sundaresan 1996; Tenorio 1997; Binmore and Swierzbinski 2000], nor auction theory [Wilson 1979; Back and Zender 1993; Wang and Zender 2002; Bikhchandani and Huang 1993] offer a constraining reason for preferring uniform to discriminatory auctions. Ausubel et al. [2014] shown that the uniform-price auction creates demand reduction incentives to bidders reversing its strategic simplicity and efficiency. This result favors small bidders over the large ones and under certain assumptions the uniform auction outperforms the discriminatory.

We encompass in our analysis the investment mandate's parameters at which investment managers abide by the objectives of investment strategies. For the asset allocation limits, we employ a budget limit in line with a risk limit. The budget limit is the available capital for investing in the new issuance (e.g. the limit in a margin account) and the risk limit is how much risk is acceptable for the investment's capital (e.g. invest only in investment-grade bonds). To our knowledge, we are the first to address these factors in a mechanism for the pricing of corporate bonds.

Prior literature in auction theory has studied little the topic of budget limits. Some works [Che 1998; Benoit and Krishna 2001] find that between the standard auction formats, the second-price auction yields lower revenues than first-price auction in the presence of financial contraint in a private-value model, and that the revenue of a simultenous ascending auction is lower than the revenue of a sequential auction. Ausubel et al. [2017] study the budget constraint as an endogenous factor and result in their experiment that the budget choices yield higher revenues and efficiency for second-price auctions. Hafalir et al. [2012] proposes a mechanism for divisible goods similar to Vickrey's with a good revenue outcome and optimality properties in which it is weakly dominated if budgets or values are understated. Dobzinski et al. [2012] show that when budgets are public information the "clinching auction" of Ausubel [2004] is individual-rational and dominant strategy incentive compatible.

We include in our model the secondary market as an exogenous random variable and we assume that all investors have the sole purpose of resale. Ex-ante, all investors have a private valuation for the bond, based on signals received for the expected equilibrium yield on secondary market. Thus, at the time of the auction the exact value of the bond, i.e. the issuance's yield, is unknown. Theoretical research on uniform-price auctions in treasury bills markets with resale have shown that the expected revenues for the auctioneer are higher versus discriminatory when there is an equilibrium [Bikhchandani and Huang 1989]. Additionally, uniform auctions favor higher information release which reduces uncertainty before the auction [Nyborg and Sundaresan 1996].

The yield of the issuance, henceforth named as the *stop-out yield*, is a market-clearing yield at the point where aggregate demand equals the full face value of the issuance, and it is defined by the *first rejected bidding schedule*. The intuition is that investors want to acquire a share of the face value at the highest possible yield to maximize their return in the secondary market.

Interestingly, we result that the number of participating investors always affects inversely the bidding strategy. This also applies to a constant factor which measures the effective demand for the bond at different yield levels. On the other side, the strategy power of budget limits is positively related to the bidding strategy. Finally, it seems that the maximum spread that an investor can earn from the resale directs his bidding.

The paper is organized as follows. The following section 2 contains a formal analysis and describes the model as a direct revelation mechanism. In the same section, we introduce the concept of risk limit. Section 3, studies bidders' incentives and provides the proof of a Bayes-Nash symmetric equilibrium for independent signals performing the respective comparative statics. The last section 4, compares the outcomes from previous sections. All proofs are expanded on the Appendix.

2 Model

2.1 Preliminaries

There is a single unit of perfectly divisible bond for sale with a face value equal to one and none of the participating investors in the auction can bid for the full face value of the bond.

There are n competitive bidders with active bond strategies[7] defined as a finite set $\mathcal{I}=\{0, 1, 2, 3 \ldots n\}$, with $n \geq 3$. Also, we consider that competitive bidders are risk-neutral, each acting to maximize their expected utility.

The equilibrium yield of the secondary market is an unknown random vari-

[7]Maximize their returns by taking advantage of expected changes in the curves.

able, $r^s \in [\underline{r},\, \bar{r}]$, with a cumulative distribution $H(r^s)$, that is common knowledge to all investors. Also, it is fully supported by a density function $h(r^s) > 0$, $\forall r^s \in [\underline{r}, \bar{r}]$. The expectation over the secondary market is denoted as $\mathbb{E}[r^s]$.

Each bidder i has upper-bound bidding, defined as the *budget limit* $c_i \in [\underline{c}, \bar{c}]$, as well as, a *risk limit* $\ell_i \in [\underline{r}, \bar{r}]$ stipulated by the investment mandate. Each bidder i has a type $\tau_i = (c_i, \ell_i)$, with $\tau \in \mathcal{T}$, which attributes bidders' preferences $\mathcal{T} := [\underline{c}, \bar{c}] \times [\underline{r}, \bar{r}]$ to the eligible real intervals.

Each type τ_i, is i.i.d. to a continuous joint cumulative function $F(\tau) = F_{c\ell}(c, \ell)$ commonly known to all bidders, fully supported by $f(\tau) > 0$.

All information for the bidding strategies $\tau_{-i} := \tau_1, \ldots, \tau_{i-1}, \tau_{i+1}, \ldots \tau_n$ is summarized in a joint cumulative function $G(r^s, \tau_{-i}) = H(r^s) \times F(\tau_{-i})$, fully supported by the density function $g(r^s, \tau_{-i}) > 0$. Additionally, bidders receive a private signal s_i about the actual private value of the bond and other bidders' preferences. This information is embedded in a signal $s_i \in \mathcal{S}$, where $\mathcal{S}$ is the signal space with an infinite number of elements that allow each bidder's value to be a general function of all the signals.

The strategy of each bidder i is a bid schedule, such as

$$b_i(r, s_i|\tau_i) : \mathcal{S} \times [\underline{r}, \bar{r}] \to [0, 1] \tag{1}$$

defined on the signals' space $\mathcal{S}$ and a domain of eligible yields $[\underline{r}, \bar{r}] \in R_+^*$ defined by the auctioneer. Each schedule specifies the quantity demanded at a specific yield based on the different realizations of private signals for the secondary market. Bid schedules are assumed to be continuously differentiable to the yield r and an increasing continuous function in the budget c and risk ℓ limits.

The issuance is produced through a mechanism $(\alpha, \hat{r})$ consisting of two components: an allocation rule α and a payment rule $\hat{r}$. The allocation is the outcome of an increasing continuous function which takes bidding strategies and parcels outs the issuance to each bidder. The payment rule $\hat{r}$ is the *stop-out yield*, common for all winners as resulted from uniform-auction format. In other words, if $b(\cdot)$ is a strategy profile for each type, then α_i is a fraction on the issuance that bidder i acquires paying $\hat{r}$.

For a strategy profile $b(\cdot)$ the payoff function of a risk-neutral bidder i given the observed signal $s_i \in S$ is:

$$\mathbb{E}_{(r^s,\tau)}[\pi_i(b|s_i)] = \mathbb{E}_{\tau|s_i}\left[\left(\hat{r}(b) - \mathbb{E}[r^s]\right)\alpha_i(b)\right]. \tag{2}$$

2.2 Market Mechanism

In this section, we will elaborate on the mechanism that produces the outcomes of the auction. The process starts with the simultaneous submission of bids. Following the uniform pricing rule [Bikhchandani and Huang 1993; Wang and Zender 2002; Krishna 2010], a step function re-indexes individual bidding schedules till the size of the issuance is fully covered.

After the auction is completed, bidders from $1, \ldots, j-1$, are called full winners and from $j+1\ldots,n$ are called losers. A cutoff bidder $j+1$ with a bid schedule $b_{j+1}(r_{j+1}, q_{j+1})$, defines the *stop-out yield* $\hat{r}$ and is the first of losers.

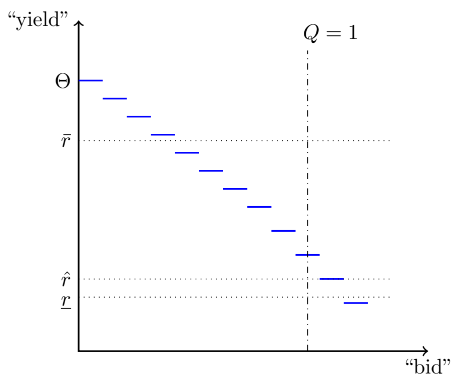

Figure 1: Minimum market-clearing yield displayed over aggregate inverse demand curve

In the case of excess demand, $D(b) > 1$, there is a bidder j called partial winner where his demand might be partially satisfied at point of *stop-out yield.* [Hafalir et al. 2012].

Definition 1 *Winning bidders receive the stop-out yield $\hat{r}$, defined as the highest losing bid in a magnitude set by the pseudo-player, where*

$$\hat{r} = min\{r \in [\underline{r}, \bar{r}] \big| D(b) \geq 1\} \tag{3}$$

and

$$r = \begin{cases} \Theta - \theta D(b) & , \textit{with } b, \theta > 0 \textit{ and } \Theta > \theta D(b) \\ \\ 0 & , \textit{otherwise} \end{cases} \tag{4}$$

where $D(b) = \sum_{i=1}^{n} b_i(\cdot|\tau_i)$ *for* $n \geq 2$.

Equation (4) is the inverse demand function of the issuance and is plotted in figure (1). The parameter Θ denotes the opportunity cost for the issuers from choosing other sources of funding (e.g. the rate of syndicated loans), while θ is a sensitivity factor which captures the movement of yield towards a change on the aggregate bids $D(b)$. In our analysis the parameter θ remains symmetric for all bidders which means that all of them have an equal market power over the yield's structure.

Here below, we define an allocation rule that specifies how the asset is allocated in a way that no bidder gets more than his demand curve i.e. bid schedule [Kremer and Nyborg 2007].

Definition 2 *An allocation rule is a mapping from the set of bid schedules' profiles* $b(\cdot)_{i=1}^{n}$ *to non-negative allocations* α*, such that* $D(b) \geq 1$*. Non-winners receive* $\{\alpha\}_{j+1}^{n} = 0$*,* $\forall j \in \mathcal{I}$ *and there is the partial winner* j *where* $a_j = \omega$ *such as* $\omega \in (0, c_j]$*, for* $c_j > 0$*.*

In other words, bidders demand a fraction of the issuance equal to their budget limit normalizing their opportunity cost to zero (i.e. they choose a risk-free rate). If they are among the full winners (bidders $1, \ldots, j-1$), they receive an allocation equal to their full budgets. The partial winner (bidder j) receives part of his budget equal to ω, leaving an unused budget.

2.3 The concept of risk limits

In this section, we will clarify the notion of risk limits, since the evaluation of the payoff is only meaningful on a risk-adjusted basis and this creates limitations in investment decisions.

The intuition behind this notion is that each asset manager complies to set of instructions or agreed constraints to carry out the management of investor's wealth. For instance, the investment mandate of funds (such as pension funds, mutual funds, ETFs, etc.) due to its idiosyncratic structures differentiate their investment strategy from retail investors. This means that the asset manager has to adhere to more rigorous guidelines which might limit the fund's ability to grab opportunities outside mandates. Baghai et al. [2019] perform a textual analysis of mutual funds' mandates and identify that credit ratings play a crucial role. The mandates require investments in investment-grade securities, fixed minimum ratings, or certain rating agencies.

To simplify our analysis we assume that bidders' mandates are horizontal and require only investment-grade bonds. Also, all bidders truthfully report their types through budget limits. We focus for tractability reasons on a direct revelation mechanism in which bidders reflect their budgets directly in their bids,i.e. the bid equals to the budget limit [Hafalir et al. 2012; Dobzinski et al. 2012].

Example. Now let us assume a bidder i with a budget constraint c_i^ℓ which takes values between $\underline{c_i} = 0$ and $\bar{c}_i$. As the demand schedule is an increasing continuous function in each bidder's budget then for $c_i^\ell < \bar{c}_i$ we have $b(\cdot|c_i^\ell) < b(\cdot|\bar{c}_i)$. By Definition (1) the inverse demand function of bidder i is given in Figure 2. The interval $[\underline{r}, \bar{r}]$ denotes the domain of eligible yields as defined by the issuer, and for simplicity, we assume that the lower bound $\underline{r}$ equals to the risk-free rate. Based on the instructions of investment mandate, bidder i will invest a budget c_i^ℓ for an acceptable risk level r_i^ℓ (e.g. bonds with at least BB^+ credit rating). In figure (2), this corresponds to point L. For the lower level of risks (e.g. a credit rating above A^+) the bidder will increase his bid b by allocating more funds, extending the budget constrain on the upper bound $\bar{c}_i$. Yet, if c_i^ℓ reaches $\bar{c}$ the bidder becomes indifferent to investing in the bond and the risk-free rate (point M). In reverse, for a higher risk limit which approaches to $\bar{r}$, the bid b will be lower and the budget will narrow close to $\underline{c_i} = 0$.

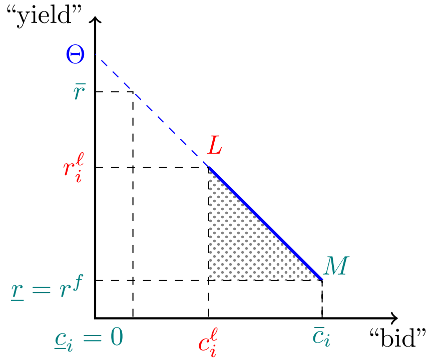

Figure 2: Mapping of budget and risk constraints on the inverse demand curve.

3 Existence of symmetric equilibrium

In this section, we prove the existence of symmetric Bayesian Nash equilibrium (see A.1). Under the assumption that all bidders choose the same strategy b^*, we examine the auction from the bidder's i point of view. The analysis from other bidders' standpoints are symmetric. In a set of strategies $b_i^*(\cdot|\tau_i)$ of n bidders, bidder i maximizes the expected payoff for different realizations of signals. On this basis of equilibrium, information updated through the signal space is the same and does not affect the outcome. Bidders privately observe the same signals before bid submission.

Definition 3 *For each strategy $b_i \in \mathcal{B}$, where $\mathcal{B}$ is the space of all strategies, there is an optimal strategy profile $b^* = (b_i^*, b_{-i}^*)$, which maximizes the expected payoff for all i, over the joint distribution $G(r^s, \tau)$ and the signal space $\mathcal{S}$. That is, for pure strategies for bidder i:*

$$\mathbb{E}_{(r^s,\tau)}[\pi_i(b_i^*, b_{-i}^*|s_i)] \geq \mathbb{E}_{(r^s,\tau)}[\pi_i(b_i, b_{-i}^*|s_i)]$$

Bidders types are independent and identically distributed in a probability function that is common knowledge to everyone and we assume that risk limit ℓ^* is symmetric and common to all.

For our analysis, we assume that $y = y^{n-1}$ is a random variable that attributes the type profiles $(n-1)$ remaining bidders, and $f_{y|\tau_i}$ denotes the conditional density function of y given τ_i. Bidder i knows his type τ_i and that the highest value component-wise in y is τ.

The expected profit of bidder i is given by:

$$\mathbb{E}(\pi_i) = \alpha_i \int_{\mathbf{c}^\ell}^{\bar{\mathbf{c}}} \Big[\hat{r}\big(b_i(\tau_i), b_{-i}(y)\big) - \mathbb{E}[r^s]\Big] f_{y|\tau_i} dy \tag{5}$$

where α_i is the allocation rule (Definition 2),where $\bar{\mathbf{c}} = \max_{j\in N/\{i\}} \bar{c}_j, \mathbf{c}^\ell = \max_{j\in N/\{i\}} c_j^\ell$ respectively.

Thus, his decision problem is to choose a bid b to solve

$$\max_{b^*} \mathbb{E}\big[\pi_i(b_i^*, b_{-i}^*|s_i)\big]$$

If b_i^* solves this problem, then the strategy b_i^* it is the best reply to $b_{-i} \dots b_n$. If each b in an $n-tuple$ of strictly increasing and differentiable strategies is a best reply to the $(n-1)$ strategy profiles, the $n-tuple$ is an equilibrium point.

Theorem 1 *The $n-tuple$ $(b^*, \dots, b^*)$ is a symmetric Nash equilibrium under uniform-price auctions, when bidders follow the same bidding strategy concerning their budget and risk limits. For $\rho(c^*) = \frac{\alpha_i'(c^*)}{\alpha_i(c^*)}$ and $\xi = \dfrac{\theta}{[\Theta - \mathbb{E}[r^s]]}$ the bidding strategy is*

$$b^*(c) = \frac{\displaystyle\int_{\mathbf{c}^\ell}^{\bar{\mathbf{c}}} \rho(c^*)\, dy}{\xi\, n} \ . \tag{6}$$

Proof. See the Appendix (A.1). □

To explain further the result of equation (6) in Theorem 1, the factor $\rho(c^*)$ captures the relative rate of change of the allocation rule α_i at budget c^*. Alike the rational of symmetric Cournot oligopoly, the more the competitive bidders the lower the equilibrium strategy.

Corollary 1 *In the equilibrium, bidders' individual demand b^* for the bond has an inverse relation with θ which is their strategic power over the yield, and it is* *directly related with the maximum anticipated return $\Theta - \mathbb{E}[r^s]$ from investing in the bond.*

Proof. Results immediately from equation (6). $\square$

Corollary 2 *Suppose that the budget limits are an increasing sequence $(c_k^\ell)_{k\in N}$, then if $\lim_{c_k^\ell \to \bar{c}} \int_{\mathbf{c}^\ell}^{\bar{\mathbf{c}}} \rho(c^*)\, dy = 0$ and, $\lim_{\bar{c} \to c_k^\ell} \int_{\mathbf{c}^\ell}^{\bar{\mathbf{c}}} \rho(c^*)\, dy = 0$, the symmetric equilibrium strategy b^* equals to zero.*

Proof. The proof results immediately from Equation (6) and Figure 2. $\square$

Proposition 1 *Symmetry across bidders implies that the yield remains unaffected by the number of participating bidders and the sensitivity factor θ.*

Proof. In the symmetry $D(b) = nb(c^*)$. By substitution of equation (6) in (4) the symmetric equilibrium yield is $\hat{r} = \Theta - \big(\int_{\mathbf{c}^\ell}^{\bar{\mathbf{c}}} \rho(c^*)\, dy\big)\big(\Theta - \mathbb{E}[r^s]\big)$ $\square$

Proposition 2 *In a uniform-price auction the equilibrium bidding strategy,ceteris paribus, is: (1) increased if the risk limit is increased, (2) decreased if the risk limit is decreased.*

From equation (6) and figure (2), it is evident that, ceteris paribus, (1) an increase in the risk limit from r^ℓ to $\bar{r}$ increases the lower bound of the integral with $\int_{\mathbf{c}^\ell}^{\bar{\mathbf{c}}} \rho(c^*)\, dy < \lim_{c_k^\ell \to \underline{c}} \int_{\mathbf{c}^\ell}^{\bar{\mathbf{c}}} \rho(c^*)\, dy$, and (2) for a decrease of r^ℓ to $\underline{r}$, the budget limit c^ℓ converges to $\bar{c}$. $\square$

4 Conclusion

This study attempts to apply auction theory to the pricing of corporate bonds. The model is consistent with the presence of a secondary market and bidders' budget and risk limits. These three aspects capture the risk profile of corporate bonds.

A symmetric Bayes-Nash equilibrium exists when the secondary market is statistically independent of bidders' types. The bidding strategy is decreased by the number of participating bidders and by the strategic power that each bidder acquires over the market-clearing yield. The bidding strategy is directly related to investment mandates. If stringent mandates are followed, the bidding will be reduced resulting in a higher issue rate.

We have modeled the opportunity cost of the issuer for other sources of lending and the anticipated equilibrium yield in the secondary market in which bidders can resale the bond.

Finally, we are considering to extent our research in parametric copula families, to better evaluate the dependency strength between bidders' types and the secondary market.

A Appendix

A.1 Proof of symmetric equilibrium

The expected profit from (5) can be rewritten as

$$\begin{aligned}\mathbb{E}(\pi_i) &= \alpha_i \int_{\underline{c}}^{c^\ell} \hat{r}(b_i(c_i), b_{-i}(y)) f(y|c^*)dy - \alpha_i \mathbb{E}[r^s] \int_{c^\ell}^{\bar{c}} f(y|c^*)dy \\ &= \alpha_i \int_{c^\ell}^{\bar{c}} \hat{r}(b_i(c_i), b_{-i}(y)) f(y|c^*)dy - \alpha_i \mathbb{E}[r^s] F(\bar{c}) \qquad (7)\end{aligned}$$

We integrate by parts the integral $\int_{c^\ell}^{\bar{c}} \hat{r}(b_i(\tau_i), b_{-i}(y)) f(y|c^*)dy$ on the right hand side of equation (7). By the continuity property of the distribution F the probability of CDF to bid below the budget limit, for $c \leq c^\ell$ equals zero.

Since, no single bidder can buy the total issuance, $\hat{r}(b_i(\tau_i), b_{-i}(c^\ell, \ell)) = \hat{r}(b_i(\tau_i), 0)) = 0$. Substituting equation (5) can be re-written:

$$\mathbb{E}(\pi_i) = \left[\hat{r}(b_i(\tau_i), b_{-i}(y))F_{\bar{c}|c^*} + \int_{\bar{c}}^{c^\ell} \hat{r}'(b_i(\tau_i), b_{-i}(y))F(y|c^*)dy - \mathbb{E}[r^s]F(\bar{c})\right]\alpha_i$$

$$\text{s.t.} \int_{\bar{c}}^{c^\ell} \hat{r}'(b_i(\tau_i), b_{-i}(y))F(y|c^*)dy \leq 0$$

To be a symmetric Bayesian Nash equilibrium, it is necessary that the first-order conditions be zero. In this case, the equilibrium spread $\int_{\bar{c}}^{c^\ell} \hat{r}'(b_i(\tau_i), b_{-i}(y))F(y|c^*)dy$ caused by the strategic behavior equals zero, because all bidders have the same type.

$$0 = \frac{\partial \mathbb{E}(\pi_i)}{\partial c_i}\Big|_{(c_i, l_i)} \tag{8}$$

$$= \left[\left[\hat{r}(b_i(c_i), b_{-i}(\bar{c}))F_{\bar{c}|c^*} + \int_{\bar{c}}^{c^\ell} \hat{r}'(b_i(\tau_i), b_{-i}(y))F(y|c^*)dy - \mathbb{E}[r^s]F(\bar{c})\right]\alpha_i(c^*)\right]'$$

Because of the symmetry all bidders share the same type c^*, so re-writing the first-order conditions:

$$= \alpha_i'(c^*)\hat{r}(b_i(c^*), b_{-i}(c^*))F(c^*) + \alpha_i(c^*)\hat{r}'(b_i(c^*), b_{-i}(c^*))F(c^*) - \alpha_i'(c^*)\mathbb{E}[r^s]F(c^*)$$

We substitute (4) to (8) and for simplicity reasons we denote $\rho(c^*) = \frac{\alpha_i'(c^*)}{\alpha_i(c^*)}$. Thus, we result:

$$\rho(c^*)\,[\Theta - n\,\theta\,b_i(c^*)]F\,(c^*) + [-\theta\,n\,b_i'(c^*)]\,F(c^*) - \rho(c^*)\,\mathbb{E}[r^s]F(c^*) = 0$$

Denominating with $(-\theta n F_c(c^*))$ and by substitution of $\xi = \dfrac{\theta}{[\Theta - \mathbb{E}[r^s]]}$, the solution is a first-order non-homogeneous differential equation:

$$b_i'(c^*) + \rho(c^*)\, b_i(c^*) = \frac{\rho(c^*)}{\xi\, n} \tag{9}$$

$$b_i(c^*) = \mathrm{e}^{-\int_{c^\ell}^{\bar{c}} \rho(c^*)\, dc} \int_{c^\ell}^{\bar{c}} \mathrm{e}^{\int_{c^\ell}^{\bar{c}} \rho(c^*)\, dc}\, \frac{\rho(c^*)}{\xi\, n}\; dc \, + \, \Gamma \tag{10}$$

where Γ is an arbitrary constant. Applying the Fundamental Theorem of Calculus for $c^* = 0$,the arbitrary constant equals zero.

By substitution $\rho(c^*)$ is the differentiation of natural log $\alpha(c^*)$. Thus, equation (10) can be rewritten as:

$$b_i(c^*) = \mathrm{e}^{\ln \alpha(c^*)\big|_{\bar{c}}^{c^\ell}} \int_{c^\ell}^{\bar{c}} \mathrm{e}^{\ln \alpha(c^*)\big|_{c^\ell}^{\bar{c}}} \left(\frac{\rho(c^*)}{\xi\, n} \right) dy$$

$$b_i(c^*) = \frac{\mathrm{e}^{\ln \alpha(c^\ell)}}{\mathrm{e}^{\ln \alpha(\bar{c})}} \int_{c^\ell}^{\bar{c}} \frac{\mathrm{e}^{\ln \alpha(\bar{c})}}{\mathrm{e}^{\ln \alpha(c^\ell)}} \left(\frac{\rho(c^*)}{\xi\, n} \right) dy$$

$$b^*(c^*) = \frac{\displaystyle\int_{c^\ell}^{\bar{c}} \rho(c^*)\, dy}{\xi\, n}$$

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